Pionic Decays of Hyperons and Universal Weak Coupling*

MASAO SUGAWARA AND TETSURO SAKUMA†
Physics Department, Purdue University, Lafayette, Indiana
(Received 6 February 1964)

The pionic decays of hyperons are analyzed in terms of the pole approximation of Feldman, Matthews, and Salam and the preceding paper. It is found that all the available experimental data are consistent with the universal weak coupling scheme, provided all the strong coupling constants have the same magnitude as the usual pion-nucleon coupling constant. In this universal weak coupling scheme, all the (dimensionless) weak coupling constants which appear in the residues of the pole terms have the same magnitude as the weak coupling constant determined in the preceding paper for the leptonic decays of pions and kaons. It is argued that this scheme is a dispersion theoretic version of the usual V–A theory of the weak interaction. The above universal coupling scheme predicts unambiguously that the S wave dominates in $\Sigma^+ \rightarrow n+\pi^+$ and the P wave dominates in $\Sigma^- \rightarrow n+\pi^-$, and also establishes some of the relative signs of the strong coupling constants. It is shown that there appears no other universal scheme of both the strong and weak coupling constants. In particular, the above universal weak coupling scheme is not compatible with the usual unitary symmetry relations for the strong coupling constants, though this conclusion depends very sensitively on the mixing parameter in the unitary symmetry relations. The main consequences and a discussion of the above universal coupling scheme are given in the last section.

I. INTRODUCTION

In the preceding paper,[1] a dispersion theoretic approach to two-body weak decays was studied, in which the masses of particles are regarded as constants. An analyticity assumption is introduced for the invariant decay amplitudes defined off the energy-momentum shell, which are invariant functions of three invariant variables. In this approach, the invariant decay amplitudes for the leptonic decays of pions and kaons are constants and, therefore, can be regarded essentially as the weak coupling constants. It was found[1] that the weak coupling constants defined in this way are independent not only of the charged lepton being the electron or the muon, but also of the decaying particle being the pion or the kaon. In the case of the pionic decays of hyperons, the invariant decay amplitudes have three kinds of poles and cuts, each corresponding to the three invariant variables. The pole terms are identical with those assumed in the pole approximation due to Feldman, Matthews, and Salam.[2]

The purpose of the present work is to discuss the question of whether or not the above universality of the weak coupling constants can be extended to the pionic decays of hyperons. For this purpose, we assume in the present work that the invariant decay amplitudes are approximated reasonably well by the pole terms alone (referred to hereinafter as the pole approximation). We assume also that the charge independence is valid in the strong interaction and the selection rule, $\Delta I=\frac{1}{2}$, is applicable to the weak interaction. The electromagnetic interaction is ignored in the pole approximation.

We summarize the pole approximation[1,2] in Sec. II. The residues of the pole terms are expressed in terms of various strong and weak coupling constants. The dimensionless weak coupling constants are defined in Sec. III, referring to some mass units. The possible significance of these mass units is also discussed. All the available experimental data are analyzed in Sec. IV to determine whether these data are consistent with the universality, in the sense that the above dimensionless weak coupling constants all have the same magnitude as that determined for the leptonic decays of pions and kaons.[1] It turns out that this is actually the case if all the strong coupling constants also have the same magnitude as the known pion-nucleon coupling constant. There does not seem to be any other universal scheme of both the strong and weak coupling constants as long as the pole approximation is approximately valid. We summarize in Sec. V the main results of the present work and, in particular, those consequences of the above universal coupling scheme which can be tested experimentally. A theoretical discussion of this universal scheme is also given.

II. POLE APPROXIMATION

In the first order of the weak Hamiltonian, $H_W(x)$, the matrix element for a hyperon with four-momentum p to decay into a nucleon (or Λ in the Ξ decay) and a pion with four-momentum q is given by[3]

$$\langle p',q|H_W(0)|p\rangle=(i/\sqrt{2q_0})(\bar{u}(p')[F\gamma_5+F']u(p)),\quad (1)$$

where u's are free Dirac spinors, normalized as $u^\dagger u=1$, $\bar{u}(p')$ stands for $u^\dagger(p')\gamma_4$, q_0 is the relativistic energy of the pion, and the invariant decay amplitudes F and F' are dimensionless. On the left-hand side of (1) [and also in (2) and (3) below], the operator and the state vectors

---

* Work supported by the National Science Foundation.

† On leave of absence from Hokkaido University, Sapporo, Japan.

[1] M. Sugawara, preceding paper, Phys. Rev. **135**, B252 (1964).

[2] G. Feldman, P. T. Matthews, and A. Salam, Phys. Rev. **121**, 302 (1961).

[3] The units, $\hbar=c=1$, are used throughout this paper. The notation of four-momentum q is such that the space components are those of three-momentum and the fourth component is iq_0.

are in the exact Heisenberg picture in which all the strong Hamiltonian $H_S(x)$ is included.

In the pole approximation,[1,2] F and F' consist of the pole terms which correspond to the diagrams summarized in Fig. 1. The residues of these pole terms are expressed in terms of various strong and weak coupling constants. For example, the strong vertex in the first diagram for $\Lambda \to p+\pi^-$ gives rise to the pion-nucleon coupling constant g_{NN} defined by

$$g_{NN}=g_{NN}(z=m_\pi^2)\,,$$
$$\langle p_p|\,\delta H_S(0)/\delta\phi(0)\,|p_n\rangle = i\sqrt{2}g_{NN}(z)(\bar{u}(p_p)\gamma_5 u(p_n))\,, \quad (2)$$

where $z=-(p_p-p_n)^2$ and $\phi(x)$ is the pion field operator. The weak vertex in the same diagram gives rise to the weak coupling constants $a_{N\Lambda}$ and $a_{N\Lambda}'$ defined by

$$a_{N\Lambda}=a_{N\Lambda}(z=0)\,,\quad a_{N\Lambda}'=a_{N\Lambda}'(z=0)\,,$$
$$\langle p_n|H_W(0)|p_\Lambda\rangle = \{\bar{u}(p_n)[a_{N\Lambda}(z)+a_{N\Lambda}'(z)\gamma_5]u(p_\Lambda)\}\,, \quad (3)$$

where $z=-(p_n-p_\Lambda)^2$. Instead of analogous definitions for other coupling constants, the effective Hamiltonians are given below which give those coupling constants which are defined in the present work when the lowest order matrix elements are evaluated with respect to the particles joining the vertices in question. The strong effective Hamiltonian is

$$\begin{aligned} H_S=\; & ig_{NN}(\bar{N}\gamma_5\boldsymbol{\tau}N)\cdot\boldsymbol{\pi}+\{ig_{\Lambda\Sigma}(\bar{\Lambda}\gamma_5\boldsymbol{\Sigma})\cdot\boldsymbol{\pi}+\text{H.c.}\} \\ & +g_{\Sigma\Sigma}(\bar{\boldsymbol{\Sigma}}\gamma_5\times\boldsymbol{\Sigma})\cdot\boldsymbol{\pi}+ig_{\Xi\Xi}(\bar{\Xi}\gamma_5\boldsymbol{\tau}\Xi)\cdot\boldsymbol{\pi} \\ & +\{ig_{N\Lambda}(\bar{N}\gamma_5\Lambda)K+ig_{N\Sigma}(\bar{N}\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\Sigma})K \\ & \quad +ig_{\Lambda\Xi}(\bar{\Xi}\gamma_5\Lambda)K_c+ig_{\Sigma\Xi}(\bar{\Xi}\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\Sigma})K_c+\text{H.c.}\}\,, \end{aligned} \quad (4)$$

where

$$N=\begin{pmatrix}P\\N\end{pmatrix},\quad \Xi=\begin{pmatrix}\Xi^0\\ \Xi^-\end{pmatrix},\quad K=\begin{pmatrix}K^+\\K^0\end{pmatrix},\quad K_c=\begin{pmatrix}\bar{K}^0\\-K^-\end{pmatrix},$$
$$\boldsymbol{\pi}=\{(\pi^++\pi^-)/\sqrt{2},\, i(\pi^+-\pi^-)/\sqrt{2},\, \pi^0\}\,, \quad (5)$$
$$\boldsymbol{\Sigma}=\{(\Sigma^++\Sigma^-)/\sqrt{2},\, i(\Sigma^+-\Sigma^-)/\sqrt{2},\, \Sigma^0\}\,.$$

The weak effective Hamiltonian is

$$\begin{aligned} H_W=\; & a_{N\Lambda}\bar{N}\Lambda+a_{N\Sigma}(\sqrt{2}\bar{P}\Sigma^+-\bar{N}\Sigma^0)+a_{\Lambda\Xi}\bar{\Lambda}\Xi^0 \\ & +a_{\Sigma\Xi}(\bar{\Sigma}^0\Xi^0+\sqrt{2}\bar{\Sigma}^-\Xi^-)+a_{N\Lambda}'\bar{N}\gamma_5\Lambda \\ & +a_{N\Sigma}'(\sqrt{2}\bar{P}\gamma_5\Sigma^+-\bar{N}\gamma_5\Sigma^0)+a_{\Lambda\Xi}'\bar{\Lambda}\gamma_5\Xi^0 \\ & +a_{\Sigma\Xi}'(\bar{\Sigma}^0\gamma_5\Xi^0+\sqrt{2}\bar{\Sigma}^-\gamma_5\Xi^-) \\ & \qquad +a_{\pi K}(\sqrt{2}\pi^+K^--\pi^0\bar{K}^0)+\text{H.c.} \end{aligned} \quad (6)$$

In (4) and (6) (and also in all the effective Hamiltonians hereafter), the particle symbols stand for the respective field operators. The expression (4) assumes nothing but the charge independence for $H_S(x)$, and the expression (6) assumes only the selection rule $\Delta I=\frac{1}{2}$ for $H_W(x)$. All the coupling constants defined by (4) and (6) are real, if $H_S(x)$ and $H_W(x)$ are time-reversal invariant.

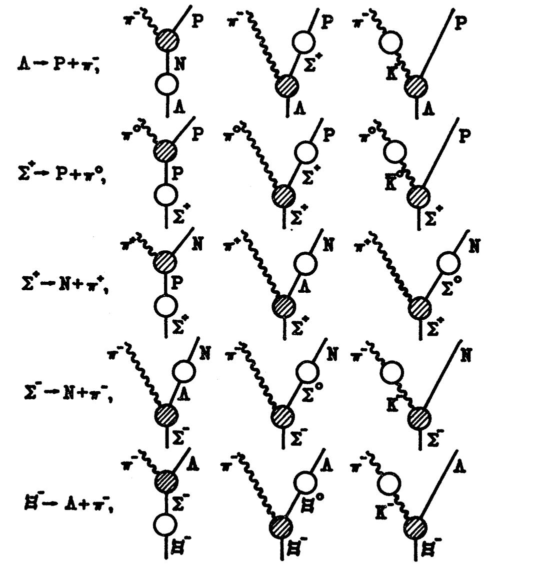

FIG. 1. Diagrams which give rise to the pole terms in F and F' defined by (1) are shown for various pionic decays of hyperons. The shaded circles stand for the strong vertices, and the open circles are the weak vertices.

In terms of the above coupling constants, F and F' in (1) are given in the pole approximation by[1]

$$\begin{aligned} F=\; & \sqrt{2}g_{NN}a_{N\Lambda}(M_n+M_\Lambda)/(M_\Lambda^2-M_n^2) \\ & +\sqrt{2}g_{\Lambda\Sigma}a_{N\Sigma}(M_{\Sigma^+}+M_p)/(M_p^2-M_{\Sigma^+}^2) \\ & +\sqrt{2}g_{N\Lambda}a_{\pi K}/(m_\pi^2-m_K^2)\,, \\ F'=\; & \sqrt{2}g_{NN}a_{N\Lambda}'(M_n-M_\Lambda)/(M_\Lambda^2-M_n^2) \\ & +\sqrt{2}g_{\Lambda\Sigma}a_{N\Sigma}'(M_{\Sigma^+}-M_p)/(M_p^2-M_{\Sigma^+}^2)\,, \end{aligned} \quad (7)$$

for $\Lambda\to p+\pi^-$ and similar expressions for other decays. The full expressions for F and F' are given at the end of the next section, by (16)–(20), in terms of the dimensionless weak coupling constants c's defined by (9).

In the pole approximation, both F and F' are real, if $H_S(x)$ and $H_W(x)$ are time-reversal invariant. One notices that $g_{\Sigma\Xi}$ does not appear in the pole approximation. Some of the relative signs of the coupling constants remain undetermined in this approximation, because F and F' either remain unchanged or change signs simultaneously when

(a) all the strong or weak coupling constants change signs,
(b) all the coupling constants which involve the kaon change signs, or (8)
(c) all the coupling constants which involve the Λ-particle change signs.

III. DIMENSIONLESS COUPLING CONSTANTS

The strong coupling constants defined by (4) are all dimensionless and the usual renormalized coupling constants for the strongly interacting particles. The weak coupling constants defined by (6) are not yet dimensionless. The baryon constants are of the dimension of energy and the boson constants are of the dimension of energy squared. We introduce in the present work the dimensionless coupling constants c by

$$\begin{aligned} a_{N\Lambda}&=(M_\Lambda-M_N)c_{N\Lambda}, & a_{N\Lambda}'&=(M_\Lambda-M_N)c_{N\Lambda}',\\ a_{N\Sigma}&=(M_\Sigma-M_N)c_{N\Sigma}, & a_{N\Sigma}'&=(M_\Sigma-M_N)c_{N\Sigma}',\\ a_{\Lambda\Xi}&=(M_\Xi-M_\Lambda)c_{\Lambda\Xi}, & a_{\Lambda\Xi}'&=(M_\Xi-M_\Lambda)c_{\Lambda\Xi}',\\ a_{\Sigma\Xi}&=(M_\Xi-M_\Sigma)c_{\Sigma\Xi}, & a_{\Sigma\Xi}'&=(M_\Xi-M_\Sigma)c_{\Sigma\Xi}',\\ a_{\pi K}&=(m_K^2-m_\pi^2)c_{\pi K}, \end{aligned} \tag{9}$$

where the masses are to be identified as those of the respective members of the charge multiplets with which the above constants a are associated in (6).

The basic motivation for the mass units in (9) is simply as follows. For example, the constant $a_{N\Lambda}$ is defined, according to (3), with respect to n and Λ and, therefore, the mass units should be either $M_\Lambda+M_n$ or $M_\Lambda-M_n$. We assume the latter because the appropriate energy unit in the decay is the mass difference.

Instead of (6), one may write the effective weak Hamiltonian as[4]

$$H_W=-c_{N\Lambda}\frac{\partial}{\partial x_\mu}(\bar{N}\gamma_\mu\Lambda)-c_{N\Lambda}'\bar{N}\left(\frac{\partial}{\partial x_\mu}\gamma_\mu\gamma_5+\gamma_5\gamma_\mu\frac{\partial}{\partial x_\mu}\right)\Lambda$$
$$-\cdots-\sqrt{2}c_{\pi K}\frac{\partial}{\partial x_\mu}\left(\frac{\partial\pi^+}{\partial x_\mu}K^- -\pi^+\frac{\partial K^-}{\partial x_\mu}\right), \tag{10}$$

where the derivative operator in the second term is to apply to the nucleon field and dots stand for the obvious terms for other baryon vertices. One observes that there are no mass factors in (10) and this expression is constructed primarily in terms of vectors and pseudovectors, whereas the expression (6) is written in terms of scalars and pseudoscalars. One also notices that the vectors in (10) are the usual weak vector currents. For this reason, one may call the weak coupling constants c the weak vector or pseudovector coupling constants.

The effective weak Hamiltonian, valid in the same sense as (6) and (10) in the leptonic decays of pions and kaons, can be written as

$$\begin{aligned} H_W=&-i(c_{\pi^-}/m_\pi)(\bar{\psi}\gamma_\mu(1+\gamma_5)\psi_\nu)(\partial\pi^-/\partial x_\mu)\\ &-i(c_{K^-}/m_K)(\bar{\psi}\gamma_\mu(1+\gamma_5)\psi_\nu)(\partial K^-/\partial x_\mu)+\text{H.c.}, \end{aligned} \tag{11}$$

where ψ and ψ_ν are the field operators for the charged lepton and the neutrino, respectively. It was found in the preceding paper[1] that the dimensionless weak coupling constants c_{π^-} and c_{K^-} have the same magnitude, that is,

$$c_{\pi^-}=c_{K^-}=1.50\times10^{-7}. \tag{12}$$

[4] This was pointed out to the authors by K. Nishijima (private communication).

The effective Hamiltonian (11) is constructed also primarily in terms of vectors and pseudovectors. Therefore, the c's in (11) may be called also the weak vector (or pseudovector) coupling constants.

If one assumes a simpler effective Hamiltonian

$$H_W=-c_{N\Lambda}\frac{\partial}{\partial x_\mu}(\bar{N}\gamma_\mu\Lambda)\pm c_{N\Lambda}'\frac{\partial}{\partial x_\mu}(\bar{N}\gamma_\mu\gamma_5\Lambda)$$
$$-\cdots-\sqrt{2}c_{\pi K}\frac{\partial}{\partial x_\mu}\left(\frac{\partial\pi^+}{\partial x_\mu}K^- -\pi^+\frac{\partial K^-}{\partial x_\mu}\right), \tag{13}$$

instead of (10), the dimensionless coupling constants defined by (13) are related to the a's in (6) by

$$\begin{aligned} &a_{N\Lambda}=(M_\Lambda-M_N)c_{N\Lambda}, \quad a_{N\Lambda}'=\pm(M_\Lambda+M_N)c_{N\Lambda}',\\ &\cdots,\\ &a_{\pi K}=(m_K^2-m_\pi^2)c_{\pi K}. \end{aligned} \tag{14}$$

If the dimensionless coupling constants are defined by

$$\begin{aligned} &a_{N\Lambda}=(M_\Lambda+M_N)c_{N\Lambda}, \quad a_{N\Lambda}'=\pm(M_\Lambda-M_N)c_{N\Lambda}',\\ &\cdots,\\ &a_{\pi K}=(m_K^2+m_\pi^2)c_{\pi K}, \end{aligned} \tag{15}$$

one does not find any simple effective Hamiltonian of the vector type.

As long as the weak coupling constants are regarded as adjustable parameters, the mass units in (9), (14), and (15) are all equivalent. However, if one assumes that the dimensionless coupling constants c have all the same magnitude, the consequence of this universality depends critically on the respective mass units, as is seen in the next section.

Finally the full expressions for F and F' in (1) are summarized in terms of the dimensionless coupling constants c defined by (10).

For $\Lambda\to p+\pi^-$:

$$F=\sqrt{2}g_{NN}c_{N\Lambda}-\sqrt{2}g_{\Lambda\Sigma}c_{N\Sigma}-\sqrt{2}g_{N\Lambda}c_{\pi K},$$

$$F'=-\left(\frac{M_\Lambda-M_n}{M_\Lambda+M_n}\right)\sqrt{2}g_{NN}c_{N\Lambda}' -\left(\frac{M_{\Sigma^+}-M_p}{M_{\Sigma^+}+M_p}\right)\sqrt{2}g_{\Lambda\Sigma}c_{N\Sigma}'. \tag{16}$$

For $\Sigma^+\to p+\pi^0$:

$$F=\sqrt{2}g_{NN}c_{N\Sigma}-\sqrt{2}g_{\Sigma\Sigma}c_{N\Sigma}+\sqrt{2}g_{N\Sigma}c_{\pi K},$$

$$F'=-\left(\frac{M_{\Sigma^+}-M_p}{M_{\Sigma^+}+M_p}\right)\sqrt{2}g_{NN}c_{N\Sigma}' -\left(\frac{M_{\Sigma^+}-M_p}{M_{\Sigma^+}+M_p}\right)\sqrt{2}g_{\Sigma\Sigma}c_{N\Sigma}'. \tag{17}$$

TABLE I. Summary of theoretical and experimental values for the decay rate, w in 10^{10} sec^{-1}, and the parameters α, β, γ, defined by (21), for various pionic decays of hyperons. The theoretical values are based upon the pole approximation, (16) to (20), with the universal coupling scheme (22) and the particle masses taken from Ref. a. This approximation assumes the time-reversal invariance, which makes β vanish identically, and also $\Delta I=\frac{1}{2}$. Therefore, the table does not include the figures for $\Lambda\to n+\pi^0$ and $\Xi^0\to n+\Lambda$.

Decay mode	$\Lambda\to p+\pi^-$	$\Sigma^+\to p+\pi^0$	$\Sigma^+\to n+\pi^+$	$\Sigma^-\to n+\pi^-$	$\Xi^-\to\Lambda+\pi^-$
w, theor.	0.34	0.89	1.28	0.29	0.23
exp.	0.26[a]	0.64[a]	0.64[a]	0.63[a]	0.71[a]
α, theor.	+0.49	−0.72	0	−0.31	−0.77
exp.	+0.61±0.05[b]	−0.79±0.09[c]	−0.05±0.08[c]	−0.16±0.21[c]	−1.0 +0.33[b]
β, exp.	−0.19±0.19[d]	...	...	...	+0.68±0.27[d]
γ, theor.	+0.87	0.70	1	−0.95	+0.64
exp.	+0.78±0.04[d]	...	...	...	+0.63±0.31[d]

[a] M. Roos, Rev. Mod. Phys. **35**, 314 (1963).
[b] L. Bertanza, V. Brisson, P. L. Connolly, E. L. Hart, I. S. Mittra *et al.*, Phys. Rev. Letters **9**, 229 (1962).
[c] R. D. Tripp, M. B. Watson, and M. Ferro-Luzzi, Phys. Rev. Letters **9**, 66 (1962).
[d] Summary by F. S. Crawford, in *Proceedings of the 1962 International Conference on High-Energy Nuclear Physics at CERN* (CERN, Geneva, 1962), p. 827.

For $\Sigma^+\to n+\pi^+$:

$$F=2g_{NN}c_{N\Sigma}-g_{\Lambda\Sigma}c_{N\Lambda}-g_{\Sigma\Sigma}c_{N\Sigma},$$
$$F'=-\left(\frac{M_{\Sigma^+}-M_p}{M_{\Sigma^+}+M_p}\right)2g_{NN}c_{N\Sigma}'-\left(\frac{M_\Lambda-M_n}{M_\Lambda+M_n}\right)g_{\Lambda\Sigma}c_{N\Lambda}'-\left(\frac{M_{\Sigma^0}-M_n}{M_{\Sigma^0}+M_n}\right)g_{\Sigma\Sigma}c_{N\Sigma}'. \tag{18}$$

For $\Sigma^-\to n+\pi^-$:

$$F=-g_{\Lambda\Sigma}c_{N\Lambda}+g_{\Sigma\Sigma}c_{N\Sigma}-2g_{N\Sigma}c_{\pi K},$$
$$F'=-\left(\frac{M_\Lambda-M_n}{M_\Lambda+M_n}\right)g_{\Lambda\Sigma}c_{N\Lambda}'+\left(\frac{M_{\Sigma^0}-M_n}{M_{\Sigma^0}+M_n}\right)g_{\Sigma\Sigma}c_{N\Sigma}'. \tag{19}$$

For $\Xi^-\to\Lambda+\pi^-$:

$$F=\sqrt{2}g_{\Lambda\Sigma}c_{\Sigma\Xi}-\sqrt{2}g_{\Xi\Xi}c_{\Lambda\Xi}+\sqrt{2}g_{\Lambda\Xi}c_{\pi K},$$
$$F'=-\left(\frac{M_{\Xi^-}-M_{\Sigma^-}}{M_{\Xi^-}+M_{\Sigma^-}}\right)\sqrt{2}g_{\Lambda\Sigma}c_{\Sigma\Xi}'-\left(\frac{M_{\Xi^0}-M_\Lambda}{M_{\Xi^0}+M_\Lambda}\right)\sqrt{2}g_{\Xi\Xi}c_{\Lambda\Xi}'. \tag{20}$$

IV. COMPARISON WITH EXPERIMENTS

In the pionic decays of hyperons, experiments can determine the decay rate w and the real parameters α, β, and γ. These are expressed in terms of F and F' in (1) as

$$\begin{aligned}w&=|\mathbf{q}|[(M-M')^2-m_\pi^2](|F|^2+|\bar{F}'|^2)/8\pi M^2,\\ \alpha&=2\,\mathrm{Re}(F^*\bar{F}')/(|F|^2+|\bar{F}'|^2),\\ \beta&=2\,\mathrm{Im}(F^*\bar{F}')/(|F|^2+|\bar{F}'|^2),\\ \gamma&=(|\bar{F}'|^2-|F|^2)/(|F|^2+|\bar{F}'|^2),\end{aligned} \tag{21}$$

with

$$\bar{F}'=\{[(M+M')^2-m_\pi^2]/[(M-M')^2-m_\pi^2]\}^{1/2}F',$$

where $\alpha^2+\beta^2+\gamma^2=1$, M and M' are the masses of the decaying hyperon and the nucleon (or Λ in the Ξ decay), respectively, and $|\mathbf{q}|$ is the pion momentum in the c.m. system. In (21), $\bar{F}'$ and F are, respectively, the S-wave and P-wave amplitudes, and the asymmetry parameter α refers to the direction of the nucleon (or Λ in the Ξ decay). All the available data are summarized in Table I.

The theoretical values in Table I are computed in the pole approximation, (16) to (20), with the universal coupling scheme

$$\begin{aligned}&g_{NN}=g_{\Lambda\Sigma}=g_{\Sigma\Sigma}=g_{\Xi\Xi}=g_{N\Lambda}=g_{N\Sigma}=g_{\Lambda\Xi}=13.6,\\ &c_{N\Lambda}=c_{N\Sigma}=c_{\Lambda\Xi}=c_{\Sigma\Xi}=c_{\pi K}=c_{N\Lambda}'\\ &\qquad=c_{N\Sigma}'=c_{\Lambda\Xi}'=c_{\Sigma\Xi}'=10^{-7}.\end{aligned} \tag{22}$$

The upper figure in (22) corresponds to $g_{NN}^2/4\pi=14.8$, that is, 0.082 for the equivalent pseudoscalar-pseudovector coupling constant squared. The lower figure in (22) is roughly $1/\sqrt{2}$ of the figure in (12). This factor was introduced because $c_{\pi K}$ refers to the neutral bosons as is seen in (6) and (9), whereas the c's in (12) refer to the charged bosons. Since all the coupling constants are real in (22), β vanishes identically and is not shown in Table I.

The agreement between the theoretical and experimental values in Table I is satisfactory. One may conclude that the universal coupling scheme (22) is consistent with all the available data.

The universal coupling scheme (22) is subject to ambiguities in the relative signs of the coupling constants in (22). Some of these ambiguities are those listed in (8). However, one can show that there are no other theoretical values, besides those in Table I, which agree even qualitatively with the data, if one requires that the strong and weak coupling constants in (22) have, respectively, the same magnitudes.

The proof of the above statement is as follows: Since α for both $\Lambda\to p+\pi^-$ and $\Sigma^+\to p+\pi^0$ is quite large, F' in (16) and (17) cannot be negligibly small, that means that

$$g_{NN}=g_{\Sigma\Sigma},\quad g_{NN}c_{N\Lambda}'=g_{\Lambda\Sigma}c_{N\Sigma}'. \tag{23}$$

One then finds in (18) and (19), independently of the relative sign of $g_{\Lambda\Sigma}$ to g_{NN}, that F' for $\Sigma^- \to n+\pi^-$ becomes negligibly small, but F' for $\Sigma^+ \to n+\pi^+$ remains appreciable. Since α for $\Sigma^+ \to n+\pi^+$ is nearly zero, F in (18) must vanish, that is, that

$$g_{NN}c_{N\Sigma} = g_{\Lambda\Sigma}c_{N\Lambda}. \tag{24}$$

The observed signs of α for $\Lambda \to p+\pi^-$ and $\Sigma^+ \to p+\pi^0$ then requires that

$$g_{N\Lambda}c_{\pi K} = g_{NN}c_{N\Lambda}', \quad g_{N\Sigma}c_{\pi K} = g_{NN}c_{N\Sigma}'. \tag{25}$$

Similarly, one obtains

$$g_{\Lambda\Sigma}c_{\Sigma\Xi}' = g_{\Xi\Xi}c_{\Lambda\Sigma}', \tag{26}$$

because α for $\Xi^- \to \Lambda+\pi^-$ is quite large. If one requires for simplicity that

$$g_{\Lambda\Sigma}c_{\Sigma\Xi} = g_{\Xi\Xi}c_{\Lambda\Sigma}, \tag{27}$$

which is analogous to (26), the observed sign of α for $\Xi^- \to \Lambda+\pi^-$ determines that

$$g_{\Lambda\Xi}c_{\pi K} = g_{\Xi\Xi}c_{\Lambda\Xi}'. \tag{28}$$

It is easy to show that the relations (23) to (28) are all those which must be satisfied in order for the universal coupling scheme (22) to be consistent with the data. The theoretical values in Table I are independent of changes of signs of the coupling constants as long as the relations (23) to (28) are satisfied.

One finds in the foregoing proof some important consequences of the universal coupling scheme (22). First, this universal scheme predicts that the S wave dominates in $\Sigma^+ \to n+\pi^+$ and the P wave dominates in $\Sigma^- \to n+\pi^-$. This can be checked experimentally by determining the sign of γ, because the theoretical values of γ in Table I are almost unity in magnitudes. Secondly the above scheme predicts, in particular, that

$$g_{NN} = g_{\Sigma\Sigma}, \quad g_{N\Lambda}/g_{N\Sigma} = g_{\Lambda\Sigma}/g_{\Sigma\Sigma}, \tag{29}$$

which follows from (23) and (25). These relations are likely subject to some tests. Thirdly, F consists, in this scheme, of the kaon-pole terms alone in all the decays in Table I. In the case of $\Sigma^+ \to n+\pi^+$, there is no kaon pole and, therefore, F vanishes. Thus, the kaon pole plays a crucial role in this scheme.

In the universal coupling scheme (22), both F and F' are proportional to the numerical figures assumed in (22). Therefore, the theoretical values for α, γ, and the ratios among the decay rates are all independent of these figures in (22). A wide variation in the experimental values of α is, according to this scheme, due to casual cancellation among various pole terms. The figures for the decay rates in Table I imply that the pole approximation may be valid to some extent but cannot be very accurate, if the universal coupling scheme (22) is valid. This may be checked experimentally by determining if β is really small or not, because β vanishes identically in the pole approximation, as long as the time-reversal invariance is valid. One sees from (19) that α for $\Sigma^- \to n+\pi^-$ vanishes in this scheme, if one ignores a mass difference between Σ^0 and Λ. Therefore, a relatively large value (-0.31) for this α in Table I is due entirely to this small mass difference.

It is added that the universal coupling scheme (22) is very similar to one of the solutions discussed in Ref. 2 and also the solution discussed by Fujii.[5]

If a universal weak coupling scheme is introduced with respect to the mass units in (14), one assumes that

$$\frac{a_{N\Lambda}}{M_\Lambda - M_n} = \cdots = \frac{a_{\pi K}}{m_K^2 - m_\pi^2} = \pm \frac{a_{N\Lambda}'}{M_\Lambda + M_n} = \cdots, \tag{30}$$

where dots stand for the obvious terms. A reasonable agreement with data is obtained in this case with

$$\begin{aligned} a_{N\Lambda}/(M_\Lambda - M_n) = \cdots \approx 10^{-8}, \quad & g_{NN} \approx g_{\Lambda\Sigma} \approx g_{\Sigma\Sigma} \approx g_{\Xi\Xi} \\ & \approx \pm(g_{N\Lambda}/20) \approx \pm(g_{N\Sigma}/20) \approx (g_{\Lambda\Xi}/20) \approx 14. \end{aligned} \tag{31}$$

If one assumes

$$\frac{a_{N\Lambda}}{M_\Lambda + M_n} = \cdots = \frac{a_{\pi K}}{m_K^2 + m_\pi^2} = \pm \frac{a_{N\Lambda}'}{M_\Lambda - M_n} = \cdots, \tag{32}$$

which refer to the mass units in (15), one obtains a reasonable agreement with data with the positive sign for all the a''s in (32) and

$$\begin{aligned} a_{N\Lambda}/(M_\Lambda + M_n) = \cdots \approx 10^{-7}, \quad & g_{NN} \approx g_{\Lambda\Sigma} \approx g_{\Sigma\Sigma} \approx g_{\Xi\Xi} \\ & \approx g_{N\Lambda}/6 \approx -2g_{N\Sigma} \approx -g_{\Lambda\Xi}/5 \approx 14, \end{aligned} \tag{33}$$

or with the negative sign for all the a''s in (32) and

$$\begin{aligned} & a_{N\Lambda}/(M_\Lambda + M_n) = \cdots \approx 10^{-7}, \\ & g_{NN} \approx 2g_{\Lambda\Sigma} \approx g_{\Sigma\Sigma} \approx g_{\Xi\Xi} \approx 14, \\ & g_{N\Lambda} \approx g_{N\Sigma} \approx g_{\Lambda\Xi} \approx 0. \end{aligned} \tag{34}$$

In the above cases, the pion coupling appears to be universal except for the case of (34). However, the kaon coupling constants are too large to be reasonable in (31) and do not seem to be universal in (33) and (34). The cases (31) and (33) are also similar to some of the solutions discussed in Ref. 2. The case (34) is somewhat analogous to the solution discussed by Gupta,[6] in the sense that no kaon poles are effective.

One finds many other solutions which fit data in the pole approximation, if the coupling constants are regarded as adjustable parameters.[7] We point out, however, that the solutions are very sensitive to the data to fit. Since the experimental uncertainties are

[5] A. Fujii, Phys. Letters **1**, 75 (1962).
[6] S. N. Gupta, Phys. Rev. **130**, 1180 (1963).
[7] One of the most extensive work of this type is J. C. Pati, Phys. Rev. **130**, 2097 (1963).

quite large and, in addition, the pole approximation can hardly be very accurate, one never knows which solutions of these may be of significance.

If unitary symmetry[8] is valid regarding the strong Hamiltonian, the strong coupling constants satisfy the relations

$$\begin{aligned} g_{\Lambda\Sigma}/g_{NN}&=2(1-f)/\sqrt{3}\,, & g_{N\Lambda}/g_{NN}&=-(1+2f)/\sqrt{3}\,,\\ g_{\Sigma\Sigma}/g_{NN}&=2f\,, & g_{N\Sigma}/g_{NN}&=(1-2f)\,,\\ g_{\Xi\Xi}/g_{NN}&=-(1-2f)\,, & g_{\Lambda\Xi}/g_{NN}&=(1-4f)/\sqrt{3}\,, \end{aligned}\tag{35}$$

where the mixing parameter f is expected[9] to be around $\frac{1}{4}$ or $\frac{1}{3}$.

One can show that (35) cannot be consistent with the S wave dominating in $\Sigma^+\to n+\pi^+$ and the P wave dominating in $\Sigma^-\to n+\pi^-$. To see this, one puts F in (18) and F' in (19) equal to zero. Ignoring a mass difference between Σ^0 and Λ, one obtains

$$c_{N\Lambda}=\sqrt{3}c_{N\Sigma}\,,\quad c_{N\Lambda}'=[\sqrt{3}f/(1-f)]c_{N\Sigma}'\,.\tag{36}$$

Then, the amplitudes in (16) are written as

$$\begin{aligned} F&=(2/3)^{1/2}(1+2f)g_{NN}(c_{N\Sigma}+c_{\pi K})\,,\\ F'&=-\Delta(2/3)^{1/2}\{[3f+2(1-f)^2]/(1-f)\}g_{NN}c_{N\Sigma}'\,, \end{aligned}\tag{37}$$

and the amplitudes in (17) become

$$\begin{aligned} F&=\sqrt{2}(1-2f)g_{NN}(c_{N\Sigma}+c_{\pi K})\,,\\ F'&=-\Delta\sqrt{2}(1+2f)g_{NN}c_{N\Sigma}'\,, \end{aligned}\tag{38}$$

where $\Delta=(M_\Sigma-M_N)/(M_\Sigma+M_N)$ and small mass differences are ignored. Since the same coupling constants appear in (37) and (38), one finds that α for $\Lambda\to p+\pi^-$ and $\Sigma^+\to p+\pi^0$ can have different signs only when $1>f>\frac{1}{2}$. The lower bound for f is raised if γ for $\Lambda\to p+\pi^-$ is required to have the observed sign. A simple calculation shows that the allowed region becomes $1>f\gtrsim0.6$ if $\alpha=-0.7$ for $\Sigma^+\to p+\pi^0$, and $1>f\gtrsim0.7$ if $\alpha=-1$ for $\Sigma^+\to p+\pi^0$. This region of f can hardly accommodate the expected values.[9] We point out that the above inconsistency is essentially due to the negative sign for $g_{N\Lambda}/g_{NN}$ in (35).

One can apply the same argument to the case when the P wave dominates in $\Sigma^+\to n+\pi^+$ and the S wave dominates in $\Sigma^-\to n+\pi^-$. In this case, (38) remain the same, but those factors in (37) which include f change as follows:

$$\begin{aligned} &1+2f\to[3f-2(1-f)^2]/(1-f)\,,\\ &[3f+2(1-f)^2]/(1-f)\\ &\qquad\to-[3(1+f)-2(1-f)^2]/(1-f)\,. \end{aligned}\tag{39}$$

[8] Y. Ne'eman, Nucl. Phys. **26**, 222 (1961); M. Gell-Mann, Phys. Rev. **125**, 1067 (1962).

[9] See, for example, A. W. Martin and K. C. Wali, Phys. Rev. **130**, 2455 (1963).

Thus, α for $\Lambda\to p+\pi^-$ and $\Sigma^+\to p+\pi^0$ can have different signs only when

$$\tfrac{1}{2}>f>0.32\quad\text{or}\quad-0.14>f>-\tfrac{1}{2}\,.\tag{40}$$

If one regards the former as an allowed region, one obtains a reasonable agreement with data concerning Λ and Σ, with the values

$$\begin{aligned} f\approx0.4\,,\quad c_{N\Lambda}&\approx c_{N\Lambda}'\approx-5c_{N\Sigma}\\ &\approx-5c_{N\Sigma}'\approx-c_{\pi K}/2\,. \end{aligned}\tag{41}$$

A more careful calculation shows that the above analyses are essentially correct. Thus, one may conclude that, if the unitary symmetry relations (35) are valid with f somewhere between 0 and $\frac{1}{2}$, the P wave must dominate in $\Sigma^+\to n+\pi^+$ and the S wave must dominate in $\Sigma^-\to n+\pi^-$. One does not find in this case any simple relationship among the weak coupling constants.

However, the above conclusion should not be taken too seriously. We know that unitary symmetry[8] is not rigorous and, therefore, the relations (35) are valid only approximately. Since the consequence of the unitary relation (35) is very sensitive to f, or the details of the relations (35), even small change in (35) might change the above conclusion.

Our foregoing analyses agree only qualitatively with a recent work by Eberle and Iwao.[10] The discrepancy appears to be due to some mistakes in their work.[10]

V. SUMMARY AND DISCUSSION

The main result of the present work is that all the available data concerning the pionic decays of hyperons are consistent with the assumption that the pole approximation[1,2] is approximately valid and all the strong and weak coupling constants which appear in the residues of the pole terms have the same magnitudes as, respectively, the usual pion-nucleon coupling constant and the weak coupling constants determined for the leptonic decays of pions and kaons.[1]

The strong coupling constants g are defined by the strong effective Hamiltonian (4) and are the usual renormalized coupling constants for the strongly interacting particles. The weak coupling constants c are defined either by the weak effective Hamiltonian (6) and the mass units in (9), or by the weak effective Hamiltonian (10), whereas the weak coupling constants in the leptonic decays of pions and kaons are defined by the effective Hamiltonian (11). Since both (10) and (11) are of the vector-pseudovector type, one may regard the universal weak coupling scheme in (22) as a dispersion theoretic version of the usual $V-A$ theory of the weak interaction. One should note, however, that no specific form of the weak Hamiltonian is assumed in the present work, except that this Hamiltonian is time-reversal invariant and satisfies the selection rule, $\Delta I=\frac{1}{2}$.

[10] E. Eberle and S. Iwao, Phys. Letters **6**, 302 (1963).

There are ambiguities in the relative signs of the coupling constants in the universal coupling scheme (22), because the theoretical values in Table I are independent of any change of the signs as long as the relations (23) to (28) are satisfied. Some of the relative signs which are fixed in this scheme are those in (29).

One of the consequences of the universal coupling scheme (22) which can be tested experimentally is that the S wave dominates in $\Sigma^+ \to n+\pi^+$ and the P wave dominates in $\Sigma^- \to n+\pi^-$. An experimental determination of the sign of the parameter γ in (21) is sufficient, because the theoretical values for γ are quite large in Table I. According to this scheme, a closer agreement in the decay rates in Table I can be attained only by going beyond the pole approximation. Thus, the figures in Table I suggest that the pole approximation cannot be very accurate. This may be checked experimentally by determining if the parameter β is really small or not, because β vanishes identically in the pole approximation, as long as the time-reversal invariance is valid. The agreement for the asymmetry parameter α is better than is to be expected. One notices that α is identically zero for $\Sigma^+ \to n+\pi^+$ but is quite large for $\Sigma^- \to n+\pi^-$. This difference in α is due to a small mass difference between Σ^0 an Λ in the pole approximation, but could very well remain beyond the pole approximation, according to this scheme. Therefore, a more accurate determination of α for $\Sigma^- \to n+\pi^-$ is very desirable.

The pole approximation[1,2] allows many other solutions,[2,5–7] if the coupling constants are regarded as adjustable parameters. However, there does not seem to be any other universal scheme of both the strong and weak coupling constants, besides (22). This is quite satisfactory, because only the mass units in (9) are physically plausible, at least to the present authors. It is, however, somewhat embarrassing that the universal weak coupling scheme in (22) is not consistent with the unitary symmetry relations (35) for the strong coupling constants. However, the strong coupling constants defined in the present work refer to the full strong Hamiltonian and, therefore, do not satisfy the rigorous unitary symmetry relations (35) because unitary symmetry is not strictly valid. As is seen in the analyses in Sec. IV, the consequence of the unitary symmetry relations (35) is very sensitive to f, and thus, to the details of (35). Therefore, the above inconsistency does apply to the relations (35), but may not necessarily apply to unitary symmetry in the usual sense.

According to Pais,[11] a possibility that

$$g_{\Lambda\Sigma}=\pm g_{\Sigma\Sigma}, \quad g_{N\Lambda}=\pm g_{N\Sigma}, \quad g_{\Lambda\Xi}=\pm g_{\Sigma\Xi}, \qquad (42)$$

with either the positive sign or the negative sign in all the terms in (42), contradicts the experimental data, provided the mass difference between Λ and Σ does not affect the argument. The first two equations in (42) are formerly the same as the second equality in (29). It is pointed out, however, that the above argument[11] does not apply to the universal coupling scheme (22). This is because the coupling constants in (22) are the renormalized coupling constants, whereas those in (42) are the unrenormalized ones which appear in the strong Hamiltonian. Therefore, there is no reason why the argument of Ref. 11 should apply to those in (22).

It is often argued that the isospin $\frac{1}{2}$ for the $\pi-\Xi$ resonance at 1535 MeV would be very difficult to explain in terms of a universal strong coupling scheme such as (22), because the isospin for the corresponding $\pi-N$ resonance is $\frac{3}{2}$. However, the differences in masses and strangeness for these systems are very likely to cause large differences in the coupling of these systems with the $K-\Lambda$ and/or $K-\Sigma$ systems. In view of the complexity of the origin of the resonance, the present authors feel that one should not accept the usual argument without a detailed analysis of these resonances.

Several authors[12,13] discussed a universal weak coupling scheme in which a weak Hamiltonian of the type of (11) was extended to the pionic decays of hyperons. This scheme explains the experimental decay rates in Table I within a factor of three but fails to explain the experimental asymmetry parameters in Table I. This is to be expected from the present dispersion approach, because the weak Hamiltonian (11) may be regarded as an effective weak Hamiltonian in the case of the leptonic decays of pions and kaons, but a weak Hamiltonian of the type of (11) extended to the pionic decays of hyperons cannot have any simple significance because the pionic decays of hyperons have too complicated structures to make any simple effective Hamiltonian of the type of (11) of any use.

ACKNOWLEDGMENT

The present authors are indebted to Dr. K. Nishijima and Dr. S. P. Rosen for valuable discussions.

[11] A. Pais, Phys. Rev. **110**, 574 (1958).

[12] S. Oneda and A. Wakasa, Nucl. Phys. **1**, 445 (1956); H. Umezawa, M. Konuma, and K. Nakagawa, *ibid.* **7**, 169 (1958).

[13] S. A. Bludman and M. A. Ruderman, Phys. Rev. **101**, 910 (1956); S. A. Bludman, Nuovo Cimento **9**, 433 (1958).